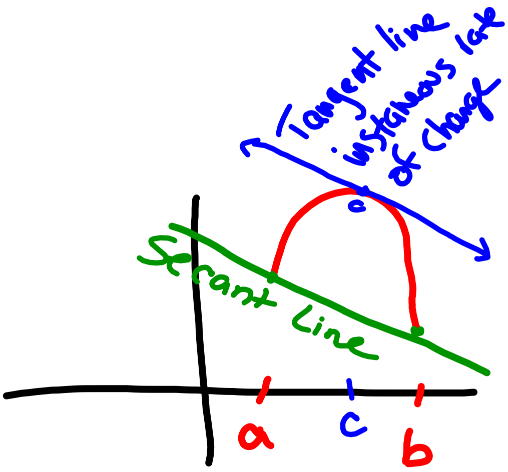
The Mean Value Theorem 2|1

If f is continuous on [a,b] and differentiable on (a,b), then there exists a number c in (a,b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$



Secant line = avg rate of change b/w a & b

Tangent line = instantaneous rate of change at "c"

For $f(x)=5-\dfrac{4}{x}$ find all values of c in the open interval (1,4) such that

$$f'(c)=\frac{\overset{=4}{f(4)}-\overset{=1}{f(1)}}{4-1}$$

f(x) is continuous & differentiable (1,4)

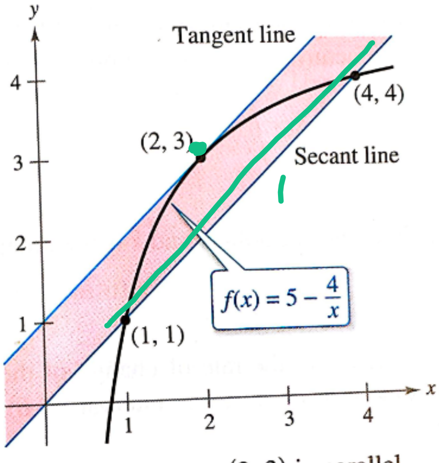
$$f'(c)=\frac{4-1}{4-1}=1$$

$$f'(c)=1$$

$$f(x)=5-\frac{4}{x}$$

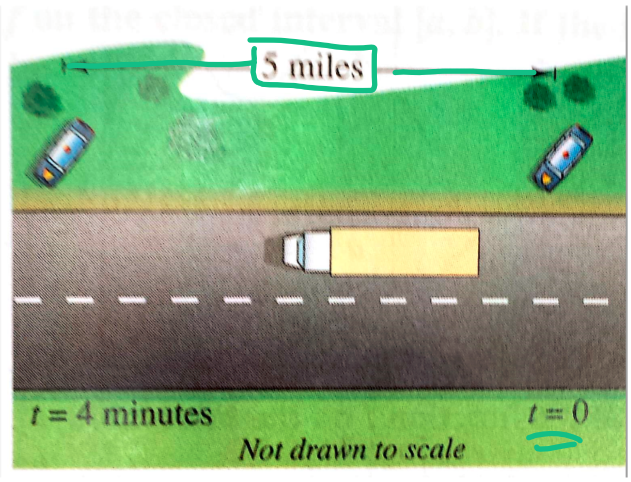
$$f'(x)=\frac{4}{x^2} \qquad \frac{4}{x^2}=1 \qquad 4=x^2 \qquad x=\pm 2$$

$$x=2$$



The tangent line at (2, 3) is parallel to the secant line through (1, 1) and (4, 4).
Figure 3.13

Two stationary patrol cars equipped with radar are 5 miles apart on a highway, as shown in the picture. As a truck passes the first patrol car, its speed is clocked at 55 mph. Four minutes later, when the truck passes the second patrol car, its speed is clocked at 50 mph. Prove that the truck must have exceeded the speed limit (of 55 mph) at some time during the 4 minutes.



t=0 when passing 1st cop

$t = 4\text{min} = \frac{4}{60} = \frac{1}{15}$ hr

$S(t)$ = distance

$S(0) = 0 \qquad S\left(\frac{1}{15}\right) = 5$ miles

avg Velocity $= \dfrac{S(\frac{1}{15}) - S(0)}{\frac{1}{15} - 0} = \dfrac{5-0}{\frac{1}{15}-0}$

75 mph